

Minimal Spanning Trees

Lecture 33

Sections 7.1 - 7.3

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- 1 Networks and Trees
- 2 Minimal Spanning Trees
- 3 Kruskal's Algorithm
- 4 Examples
- 5 Assignment

Outline

- 1 Networks and Trees
- 2 Minimal Spanning Trees
- 3 Kruskal's Algorithm
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Networks and Trees

Definition (Network)

A **network** is a connected graph.

Networks and Trees

Definition (Network)

A **network** is a connected graph.

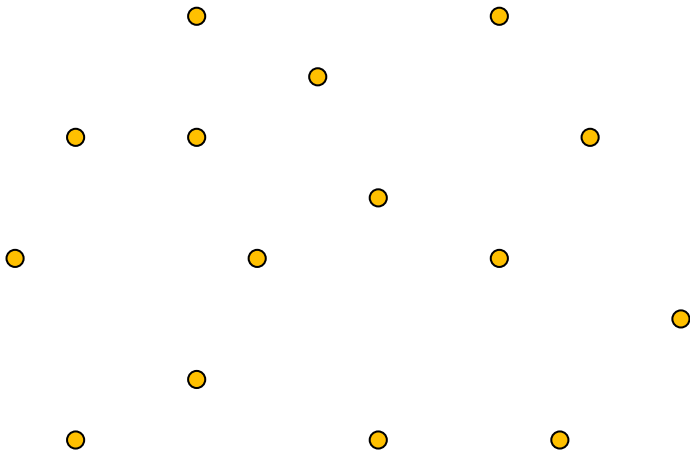
Definition (Tree)

A **tree** is a network that contains no circuits.

Properties of Trees

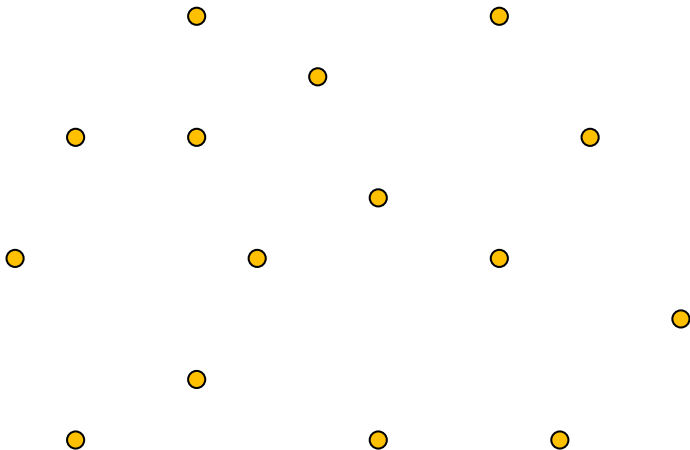
- If T is a tree with N vertices, then
 - T has exactly $N - 1$ edges.
 - If any edge of T is removed, then the resulting graph is not connected.

Properties of Trees



Add edges to connect the graph without making a circuit.

Properties of Trees



Erase edges to remove circuits without disconnecting the graph.

Properties of Trees

Theorem

A graph G with N vertices is a tree if any two of the following three properties hold.

- (1) T has exactly $N - 1$ edges.*
- (2) T is connected.*
- (3) T contains no circuits.*

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Examples

Computer ●

● Printer

Computer ●

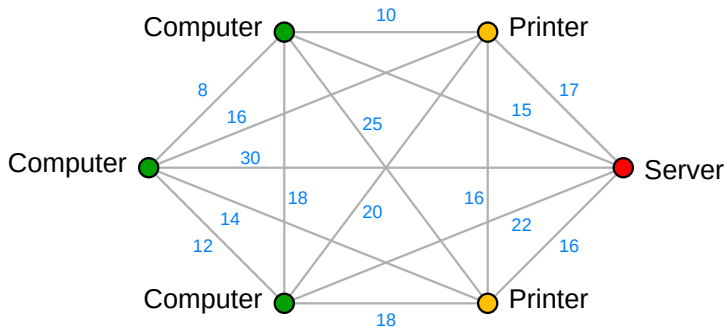
● Server

Computer ●

● Printer

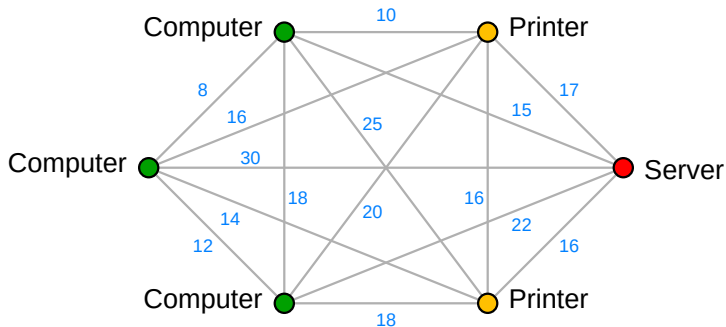
Suppose we want to connect devices in a network.

Examples



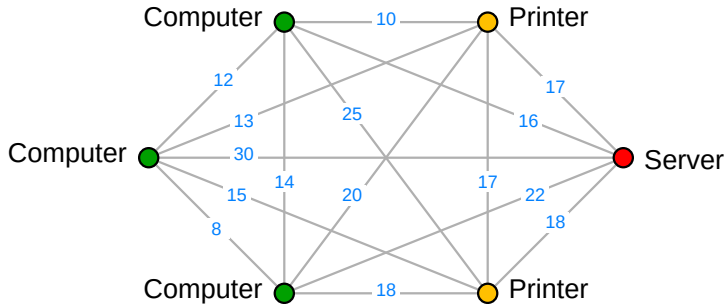
We want all the devices connected to each other.

Examples



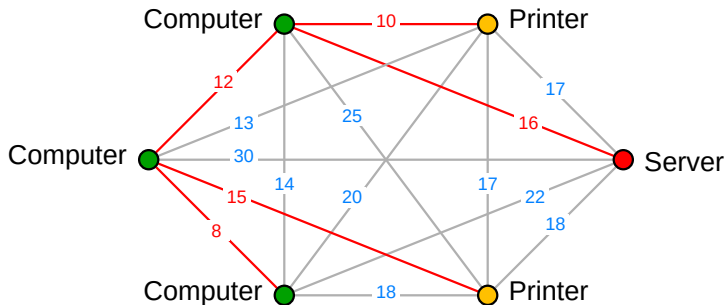
But no two devices need to be connected by more than a single path.

Examples



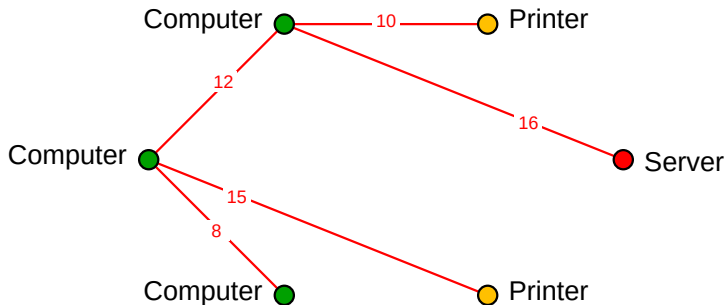
Given the distances, which lines do we keep?

Examples



The best choice is the **tree** of **minimal** total length

Examples



The total length of this tree is 61

Minimal Spanning Trees

Definition (Spanning Tree)

Given a graph G , a **spanning tree** of G is a subgraph T that is a tree and includes all the vertices of G .

Minimal Spanning Trees

Definition (Spanning Tree)

Given a graph G , a **spanning tree** of G is a subgraph T that is a tree and includes all the vertices of G .

Definition (Minimal Spanning Tree)

Given a weighted graph G , a **minimal spanning tree** of G is a spanning tree T that has the smallest total weight of all possible spanning trees of G .

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Kruskal's Algorithm

Kruskal's Algorithm

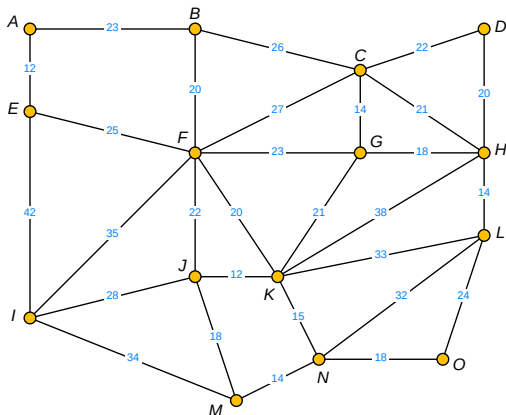
Given a graph G of N vertices, we may construct a minimal spanning tree as follows.

- (1) Include all the vertices of G in T .
- (2) Add to T the edge of minimal weight.
- (3) Repeatedly add to T edges of minimal weight of the remaining edges while being careful not to create a circuit.
- (4) Once we have added $N - 1$ edges, T is a minimal spanning tree of G .

Outline

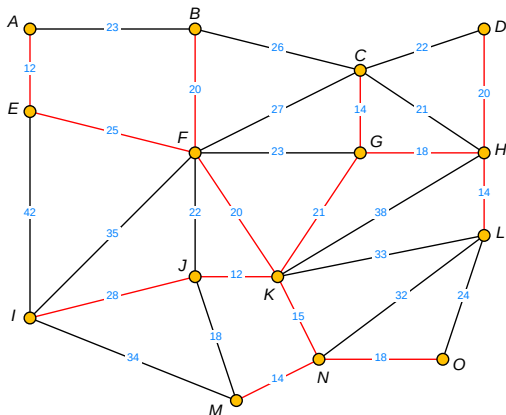
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Power Grid



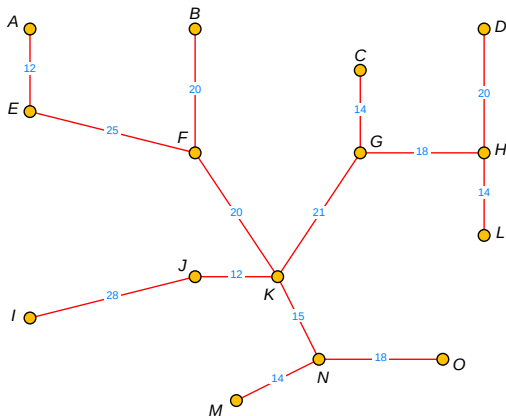
Given the possible links, find the minimal spanning tree.

Power Grid



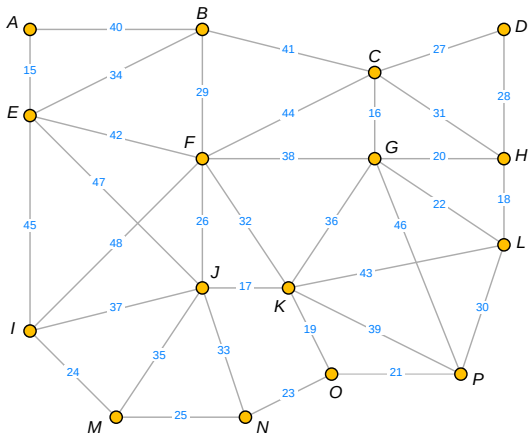
The minimal spanning tree.

Power Grid



Total weight is 251.

Practice



Find the minimal spanning tree.

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Assignment

Assignment

- Chapter 7 Exercises 1, 3, 7, 35, 37, 39, 40, 45.